IDLinearWaves

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Abstract

Provides gravitational wave solutions to the linearized Einstein equations

1 Purpose

There are two different linearized initial data sets provided, plane waves and Teukolsky waves.

2 Plane Waves

A full description of plane waves can be found in the PhD Thesis of Malcolm Tobias, *The Numerical Evolution of Gravitational Waves*, which can be found at `http://wugrav.wustl.edu/Papers/Thesis97/Thesis97.html`.

Plane waves travelling in arbitrary directions can be specified. For these plane waves the TT gauge is assumed (the metric perturbations are transverse to the direction of propagation, and the metric is traceless). In the case of waves travelling along the $z-$direction this would give the *plus* solution

$$h_{xx} = -h_{yy} = f(t \pm z), h_{xy} = h_{xz} = h_{yz} = h_{zz} = 0$$

and the *cross* solution

$$h_{xy} = h_{yx} = f(t \pm z), h_{yz} = h_{xx} = h_{yy} = h_{zz} = 0$$

This thorn implements the `plus` solution, with the waveform $f(t \pm z)$ having the form of a Gaussian modulated sine function. Now working with a general direction of propagation k we have the plane wave solution:

$$f(t,x,y,z) = A_{in} e^{-(k_i^p x^i + \omega_p (t - r_a))^2} \cos(k_i x^i + \omega t) + A_{out} e^{-(k_i^p x^i - \omega_p (t - r_a))^2} \cos(k_i x^i - \omega t)$$

and

$$\begin{aligned} g_{xx} &= 1 + f[\cos^2\phi - \cos^\theta \sin^2\phi] \\ g_{xy} &= -f \sin^2\theta \sin\phi \cos\phi \\ g_{xz} &= f \sin\theta \cos\theta \sin\phi \\ g_{yy} &= 1 + f[\sin^2\phi - cos^2\theta \cos^2\phi] \\ g_{yz} &= f \sin\theta \cos\theta \cos\phi \\ g_{zz} &= 1 - f \sin^2\theta \end{aligned}$$

The extrinsic curvature is then calculated from

$$K_{ij} = -\frac{1}{2\alpha} \dot{g}_{ij} \tag{1}$$

3 Teukolsky waves

Teukolsky waves are quadrupole wave solutions to the linearized Einstein equations. For a full description, see: PRD 26:745 (1982).

4 Comments

The extrinsic curvature is initialized assuming the initial lapse is one.