ADM

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Abstract

Spacetime evolver for the ADM variables

1 Comments

This thorn evolves the standard ADM equations, see [1]. The line element is

$$ds^2 = -(\alpha^2 - \beta^i\beta_i)dt^2 + \beta_i dt dx^i + \gamma_{ij} dx^i dx^j, \tag{1}$$

where α is the lapse, β_i the shift vector and γ_{ij} the 3-metric. Defining n to be the normal to the slice, we have the extrinsic curvature K_{ij} given by

$$K_{ij} = \frac{1}{2}\mathcal{L}_n\gamma_{ij} \tag{2}$$

where $\mathcal{L}$ is the Lie derivative.

The ADM equations then evolve the spatial three metric γ_{ij} and the extrinsic curvature K_{ij} using

$$\begin{aligned} \frac{d}{dt}\,\gamma_{ij} &= -2\alpha K_{ij}, && (3)\\ \frac{d}{dt}\,K_{ij} &= -D_iD_j\alpha + \alpha\Big(R_{ij} + KK_{ij} \\ &\quad -2K_{ik}K^k{}_j - {}^{(4)}R_{ij}\Big), && (4) \end{aligned}$$

with

$$\frac{d}{dt} = \partial_t - \mathcal{L}_\beta \tag{5}$$

and where $\mathcal{L}_\beta$ is the Lie derivative with respect to the shift vector β^i. Here R_{ij} is the Ricci tensor and D_i the covariant derivative associated with the three-dimensional metric γ_{ij}. The 4-dimensional Ricci tensor ${}^{(4)}R_{ij}$ is usually written in terms of the energy density ρ and stress tensor S_{ij} of the matter as seen by the normal (Eulerian) observers:

$${}^{(4)}R_{ij} = 8\pi\left[S_{ij} - \frac{1}{2}\,(S-\rho)\right]. \tag{6}$$

References

[1] J. York, in *Sources of Gravitational Radiation*, edited by L. Smarr (Cambridge University Press, Cambridge, England, 1979).