CactusEinstein

David Rideout

Date: 2005/10/22 18:21:44

Abstract

Set of thorns for performing simulations of Einstein's equations, which encode the dynamics of General Relativity. The package includes initial data, evolution codes, analysis tools, and various 'utility' thorns which provide infrastructure such as a time invariant conformal factor and a spatial 'mask' which can be used to put various flags on your special grid points.

1 Purpose

This document is intended to be a 'third read' quick start guide to using the `CactusEinstein` toolkit to do fully general relativistic simulations. (The first and second reads may be the QuickStart HOWTO and a skim through parts A and B of the Users' Guide, respectively.) Since it will be improved vastly by input from *first-time user* experiences, please send comments and suggestions to `users@cactuscode.org`. Better yet, ask for a cvs[1] account so that you can edit this document directly yourself!

2 Overview

The `CactusEinstein` arrangement provides a suite of thorns for doing numerical relativity simulations. The basic variables are those of the ADM formulation of Einstein's equations, namely the spatial 3-metric γ_{ij}, the lapse α, the shift β, and the extrinsic curvature K_{ij}. The 4-metric is given by

$$ds^2 = -(\alpha^2 - \beta^i\beta_i)dt^2 + \beta_i dt dx^i + \gamma_{ij}dx^i dx^j \ .$$

If γ_{ij} is the 3-metric of a spacelike Cauchy surface with normal n, then

$$K_{ij} = \frac{1}{2}\mathcal{L}_n\gamma_{ij} \ .$$

These variables are defined in the thorn `ADMBase`, and are the ones that are used to communicate the geometry to other thorns. It is not necessary to use these as the basic variables of your computation, however. All you need do is compute these quantities at each iteration in order to communicate with the `CactusEinstein` thorns.

There are a number of initial data thorns, an `ADM` thorn for evolution, and thorns which provide many other pieces of infrastructure. It is not necessary to use all of these thorns to make use of `CactusEinstein`, however. The only thorn which is necessary is `ADMBase`, since it defines the variables and parameters on which the rest of the `CactusEinstein` thorns depend.

[1] What is cvs? See the cvs appendix of the Users' Guide.

3 Matter Interface

Matter is handled in `CactusEinstein` via the fabled "`CalcTmunu`" interface.

This is documented in detail in the ThornGuide for `ADMCoupling`.

4 The Thorns

Here we give a brief description of each of the thorns contained in this arrangement.

- **ADM** evolves γ_{ij} and K_{ij} via the standard ADM equations; this thorn is semi-obselete, as it doesn't use the modern method of lines generic evolution scheme (provided by the **MoL** thorn in the **CactusBase** arrangement); it's not clear if there's any Cactus thorn which evolves the ADM equations using the method of lines[2]
- **ADMAnalysis** routines run at CCTK_ANALYSIS to compute various quantities
- **ADMConstraints** computes the $3+1$ Hamiltonian (energy) and momentum constraints
- **ADMCoupling** allows thorns to 'register' their matter field contributions to the stress energy tensor
- **ADMMacros** macros for computing various quantities which are commonly used in $3+1$ numerical relativity, such as Christoffel symbols, covariant derivatives, the Ricci tensor, etc etc; some of these support both 2nd and 4th order finite differencing
- **AHFinder** searches for apparent horizons[3]
- **CoordGauge** manages gauge quantities
- **EvolSimple** a demo evolution thorn
- **Extract** 'extracts' gravitational-wave waveforms
- **IDAnalyticBH** analytic black hole initial data
- **IDAxiBrillBH** axisymmetric Brill wave with black hole initial data
- **IDBrillData** Brill wave initial data
- **IDLinearWaves** linearized wave initial data
- **IDSimple** a demo initial data thorn, provides Minkowski space with conformal factor
- **Maximal** maximal slicing gauge condition
- **PsiKadelia** computes various Neumann-Penrose quantities
- **SpaceMask** provides a 'mask' for the spatial grid

[2]The **BSSN_MoL** thorn (in the **AEIThorns** arrangement) evolves the BSSN system of equations using the method of lines. The BSSN system uses a reparameterization of the ADM variables γ_{ij} and K_{ij} and their evolution equations, which usually gives more stable evolutions.

[3]There are also two other apparent-horizon-finder thorns in Cactus. The **TAT** arrangement has the **TGRapparentHorizon2D** thorn, and the **AEIThorns** arrangement has the **AHFinderDirect** thorn. As of late 2004, **AHFinderDirect** seems to be the only Cactus apparent horizon finder which is actively maintained, and most new work is using it.

- **StaticConformal** provides for a static conformal factor
- **TimeGeodesic** computes timelike geodesics

5 Some Useful Thorns in Other Arrangements

There are lots of other thorns in Cactus; some particularly useful ones include

CactusBase arrangement

CartGrid3D sets up the 3-D Cartesian grid

CoordBase provides an elegant and powerful way of specifying the grid size and shape, including any ghost zones

AEIThorns arrangement

Exact analytical solutions where the full 4-metric is known throughout the entire spacetime, eg. Schwarzschild, Kerr, various cosmological solutions, etc etc