IDScalarWave

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Abstract

Initial Data for the 3D Scalar Wave Equation

1 Purpose

This thorn provides different initial data for the 3D scalar wave equation.

2 Spherically Symmetric Solutions

The general spherically symmetric solution can be written

$$\Psi(r,t) = \frac{1}{r}\left(f(r+t) + g(r-t)\right) \tag{1}$$

where the functions f and g can be freely chosen.

Making the additional requirement of time symmetry at $t = 0$, forces

$$f(r) = g(r) \tag{2}$$

Thus if the solution at t=0 is given by $\phi(r)$, the general solution will be

$$\Psi(r,t) = \frac{1}{2r}\left((r+t)\phi(r+t) + (r-t)\phi(r-t)\right) \tag{3}$$

3 Gaussian

The gaussian solution is *spherically symmetric* about the origin of the Cartesian coordinate system, and is *time symmetric*. The initial profile is

$$\phi(r) = A\exp(-r^2/\sigma) \tag{4}$$

with the solution at the origin being

$$\Psi(r=0,t) = \left(1 - 2\frac{t^2}{\sigma}\right)\exp(-t^2/\sigma) \tag{5}$$

The Gaussian solution is set with the parameters

- `amplitude` $= A$
- `sigma` $= \sigma$