IDBrillData

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Date: 2002/06/04 12:50:57

Abstract

This thorn creates time symmetric initial data for Brill wave spacetimes. It can create both axisymmetric data (in a 3D cartesian grid), as well as data with an angular dependency.

1 Purpose

The purpose of this thorn is to create (time symmetric) initial data for a Brill wave spacetime. It does so by starting from a three–metric of the form originally considered by Brill

$$ds^2 = \Psi^4 \left[e^{2q}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right] = \Psi^4 \hat{ds}^2, \tag{1}$$

where q is a free function subject to certain regularity and fall-off conditions, $\rho = \sqrt{x^2+y^2}$ and Ψ is a conformal factor to be solved for.

Thorn `IDBrillData` provides three choices for the q function: an exponential form, (`IDBrillData::q_function = "exp"`)

$$q = a\,\frac{\rho^{2+b}}{r^2} e^{-\left(\frac{z}{\sigma_z}\right)^2} e^{-(\rho-\rho_0)^2}\left[1 + d\frac{\rho^m}{1+e\rho^m}\cos^2\left(n\phi+\phi_0\right)\right] \tag{2}$$

a generalized form of the q function first written down by Eppley (`IDBrillData::q_function = "eppley"`)

$$q = a\left(\frac{\rho}{\sigma_\rho}\right)^b \frac{1}{1+\left[\left(r^2-r_0^2\right)/\sigma_r^2\right]^{c/2}}\left[1 + d\frac{\rho^m}{1+e\rho^m}\cos^2\left(n\phi+\phi_0\right)\right] \tag{3}$$

and the (default) Gundlach q function which includes the Holz form (`IDBrillData::q_function = "gundlach"`)

$$q = a\left(\frac{\rho}{\sigma_\rho}\right)^b e^{-\left[\left(r^2-r_0^2\right)/\sigma_r^2\right]^{c/2}}\left[1 + d\frac{\rho^m}{1+e\rho^m}\cos^2\left(n\phi+\phi_0\right)\right] \tag{4}$$

Substituting the metric into the Hamiltonian constraint gives an elliptic equation for the conformal factor Ψ which is then numerically solved for a given function q:

$$\hat{\nabla}\Psi - \frac{\Psi}{8}\hat{R} = 0 \tag{5}$$

where the conformal Ricci scalar is found to be

$$\hat{R} = -2\left(e^{-2q}(\partial_z^2 q + \partial_\rho^2 q) + \frac{1}{\rho^2}(3(\partial_\phi q)^2 + 2\partial_\phi q)\right) \tag{6}$$

Assuming the initial data to be time symmetric means that the momentum constraints are trivially satisfied.

In the case of axisymmetry (that is $d = 0$ in the above expressions for q), the Hamiltonian constraint can be written as an elliptic equation for Ψ with just the flat space Laplacian,

$$\nabla_{flat}\Psi + \frac{\Psi}{4}(\partial_z^2 q + \partial_\rho^2 q) = 0 \tag{7}$$

If the initial data is chosen to be `ADMBase::initial_data = "brilldata2D"` then this elliptic equation is solved rather than the equation above.

2 Generating Initial Data with IDBrillData

Brill initial data is activated by choosing the `CactusEinstein/ADMBase` parameter `initial_data` to be `brilldata`, or for the case of axisymmetry `brilldata2D` can also be used.

The parameter `IDBrillData::q_function` chooses the form of the q function to be used, defaulting to the Gundlach expression.

Additional `IDBrillData` parameters for each form of q fix the remaining freedom:

- Exponential q: `IDBrillData::q_function = "exp"`
  $(a, b, \sigma_z, \rho_0) =$ (`exp_a, exp_b, exp_sigmaz,exp_rho0`)
- Eppley q: `IDBrillData::q_function = "eppley"`
  $(a, b, \sigma_\rho, r_0, \sigma_r, c) =$ (`eppley_a, eppley_b, eppley_sigmarho, eppley_r0, eppley_sigmar, eppley_c`)
- Gundlach q: `IDBrillData::q_function = "gundlach"`
  $(a, b, \sigma_\rho, r_0, \sigma_r, c) =$ (`gundlach_a, gundlach_b, gundlach_sigmarho, gundlach_r0, gundlach_sigmar, gundlach_c`)
- Non-axisymmetric part for each choice of q
  $(d, m, e, n, \phi 0) =$ (`brill3d_d, brill3d_m, brill3d_e, brill3d_n, brill3d_phi0`)

Note that the default q expression is

$$q = \texttt{gundlach\_a} \quad \rho^2 e^{-r^2}$$

`IDBrillData` can use the elliptic solvers (type LinMetric) provided by `CactusEinstein/EllSOR`, `AEIThorns/BAM_Elliptic`, or `CactusElliptic/EllPETSc` to solve the equation resulting from the Hamiltonian constraint. In all cases the parameter `thresh` sets the threshold for the elliptic solve. The choice of elliptic solver is made through the parameter `brill_solver`:

- `sor`: Understands the Robin boundary condition, additional parameters control the maximum number of iterations (`sor_maxit`).
- `bam`: `BAM_Elliptic` does not properly implement the elliptic infrastructure of `EllBase`, and the `BAM_Elliptic` parameter to use the Robin boundary condition must be set independently of `IDBrillWave::brill_bound`.

3 Notes

Thorn `IDBrillData` understands both the "`physical`" and "`static conformal`" `metric_type`. In the case of a conformal metric being chosen, the conformal factor is set to Ψ. Currently the derivatives of the conformal factor are not calculated, so that only `staticconformal::conformal_storage = "factor"` is supported.

4 References

4.1 Specification of Brill Waves

1. Dieter Brill, **Ann. Phys.**, 7, 466, 1959.
2. Ken Eppley, **Sources of Gravitational Radiation**, edited by L. Smarr (Cambridge University Press, Cambridge, England, 1979), p. 275.

4.2 Numerical Evolutions of Brill Waves

1. *Gravitational Collapse of Gravitational Waves in 3D Numerical Relativity*, Miguel Alcubierre, Gabrielle Allen, Bernd Bruegmann, Gerd Lanfermann, Edward Seidel, Wai-Mo Suen, Malcolm Tobias, **Phys. Rev. D61**, 041501, 2000.