ADMAnalysis

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Abstract

Basic analysis of the metric and extrinsic curvature tensors

1 Purpose

This thorn provides analysis routines to calculate the following quantities:

- The trace of the extrinsic curvature (trK).
- The determinant of the 3-metric ($detg$).
- The components of the 3-metric in spherical coordinates ($g_{rr}, g_{r\theta}, g_{r\phi}, g_{\theta\theta}, g_{\phi\theta}, g_{\phi\phi}$).
- The components of the extrinsic curvature in spherical coordinates ($K_{rr}, K_{r\theta}, K_{r\phi}, K_{\theta\theta}, K_{\theta\phi}, K_{\phi\phi}$).
- The components of the 3-Ricci tensor in cartesian coordinates ($\mathcal{R}_{ij}$) for $i, j \in \{1, 2, 3\}$.
- The Ricci scalar ($\mathcal{R}$).

2 Trace of Extrinsic Curvature

The trace of the extrinsic curvature at each point on the grid is placed in the grid function `trK`. The algorithm for calculating the trace uses the physical metric, that is it includes any conformal factor.

$$\texttt{trK} \equiv trK = \frac{1}{\psi^4} g^{ij} K_{ij} \tag{1}$$

3 Determinant of 3-Metric

The determinant of the 3-metric at each point on the grid is placed in the grid function `detg`. This is always the determinant of the conformal metric, that is it does not include any conformal factor.

$$\texttt{detg} \equiv detg = -g_{13}^2 * g_{22} + 2 * g_{12} * g_{13} * g_{23} - g_{11} * g_{23}^2 - g_{12}^2 * g_{33} + g_{11} * g_{22} * g_{33} \tag{2}$$

4 Transformation to Spherical Cooordinates

The values of the metric and/or extrinsic curvature in a spherical polar coordinate system (r, θ, ϕ) evaluated at each point on the computational grid are placed in the grid functions (`grr`, `grt`, `grp`, `gtt`, `gtp`, `gpp`) and (`krr`, `krt`, `krp`, `ktt`, `ktp`, `kpp`). In the spherical transformation, the θ coordinate is referred to as **q** and the ϕ as **p**.

The general transformation from Cartesian to Spherical for such tensors is

$$
\begin{aligned}
A_{rr} &= \sin^2\theta\cos^2\phi A_{xx} + \sin^2\theta\sin^2\phi A_{yy} + \cos^2\theta A_{zz} + 2\sin^2\theta\cos\phi\sin\phi A_{xy} \\
&\quad +2\sin\theta\cos\theta\cos\phi A_{xz} + 2\sin\theta\cos\theta\sin\phi A_{yz} \\
A_{r\theta} &= r(\sin\theta\cos\theta\cos^2\phi A_{xx} + 2*\sin\theta\cos\theta\sin\phi\cos\phi A_{xy} + (\cos^2\theta - \sin^2\theta)\cos\phi A_{xz} \\
&\quad +\sin\theta\cos\theta\sin^2\phi A_{yy} + (\cos^2\theta - \sin^2\theta)\sin\phi A_{yz} - \sin\theta\cos\theta A_{zz}) \\
A_{r\phi} &= r\sin\theta(-\sin\theta\sin\phi\cos\phi A_{xx} - \sin\theta(\sin^2\phi - \cos^2\phi)A_{xy} - \cos\theta\sin\phi A_{xz} \\
&\quad +\sin\theta\sin\phi\cos\phi A_{yy} + \cos\theta\cos\phi A_{yz}) \\
A_{\theta\theta} &= r^2(\cos^2\theta\cos^2\phi A_{xx} + 2\cos^2\theta\sin\phi\cos\phi A_{xy} - 2\sin\theta\cos\theta\cos\phi A_{xz} + \cos^2\theta\sin^2\phi A_{yy} \\
&\quad -2\sin\theta\cos\theta\sin\phi A_{yz} + \sin^2\theta A_{zz}) \\
A_{\theta\phi} &= r^2\sin\theta(-\cos\theta\sin\phi\cos\phi A_{xx} - \cos\theta(\sin^2\phi - \cos^2\phi)A_{xy} + \sin\theta\sin\phi A_{xz} \\
&\quad +\cos\theta\sin\phi\cos\phi A_{yy} - \sin\theta\cos\phi A_{yz}) \\
A_{\phi\phi} &= r^2\sin^2\theta(\sin^2\phi A_{xx} - 2\sin\phi\cos\phi A_{xy} + \cos^2\phi A_{yy})
\end{aligned}
$$

If the parameter `normalize_dtheta_dphi` is set to `yes`, the angular components are projected onto the vectors $(rd\theta, r\sin\theta d\phi)$ instead of the default vector $(d\theta, d\phi)$. That is,

$$
\begin{aligned}
A_{\theta\theta} &\rightarrow A_{\theta\theta}/r^2 \\
A_{\phi\phi} &\rightarrow A_{\phi\phi}/(r^2\sin^2\theta) \\
A_{r\theta} &\rightarrow A_{r\theta}/r \\
A_{r\phi} &\rightarrow A_{r\phi}/(r\sin\theta) \\
A_{\theta\phi} &\rightarrow A_{\theta\phi}/r^2\sin\theta)
\end{aligned}
$$

5 Computing the Ricci tensor and scalar

The computation of the Ricci tensor uses the ADMMacros thorn. The calculation of the Ricci scalar uses the generic trace routine in this thorn.